Exploration Guide

Maths IB Standard Level and Higher Level
Applications and Interpretations
Analysis and Approaches
(For first examination in 2021).

Essential information

The exploration in IB maths has now been standardised – with the same marking criteria used for SL Analysis and SL Applications, and a slightly different marking criteria used for both HL Analysis and HL Applications.

- Your exploration can be on any area of personal interest – as long as it incorporates mathematics commensurate with the level of the course you are taking.

- Your exploration should be 12-20 pages long

- Your exploration is worth 20% of your final grade and is awarded a mark out of 20.

- During this process you should hand in a **draft version** of your exploration to your teacher. This should be what you believe to be a finished version of your exploration. Your teacher is then allowed to fully annotate this, check your mathematics and presentation before giving you written and verbal feedback as to how to improve.

- Your teacher should provide you with a copy of (or the relevant sections from):
  "Mathematics: analysis and approaches teacher support material"
  "Mathematics: analysis and approaches guide"
  Both of these two documents have detailed advice from the IB about how to choose a topic, the full marking criteria used by teachers and moderators and a lot of useful advice for completing a successful investigation.

- On the previously used similar criteria (last examination 2020) you needed the following scores for each level:
  HL: Level 7: 17-20. Level 6: 15-16. Level 5: 12-14. Level 4: 9-11.
  SL: Level 7: 18-20. Level 6: 15-17. Level 5: 12-14. Level 4: 9-11.

The exploration marking criteria

You will be marked against the following criteria strands:

A: Communication [out of 4].

B: Mathematical communication [out of 4]

C: Personal engagement [out of 3].

D: Reflection [out of 3]

E: Use of mathematics [out of 6]

A: Communication

To achieve A4 your exploration needs to be coherent, well organised, and concise.

Coherent explorations are easy to follow with steps clearly explained and sections which link together. It should be communicated so that a fellow student in your class can understand.

Organised explorations have introductions, clear aim and rationale and a conclusion. Work is cited in-text, with a bibliography. Graphs and tables are organised and presented in the relevant place.

Concise explorations have no repetitive calculations (once you have demonstrated your method once you don't need to so numerous times again). Content included should be relevant to the exploration. You should finish your exploration by achieving your aim. It should not be more than 20 pages.

Criteria A student checklist – have you done the following?

· An introduction explaining the exploration.

· An aim outlining the purpose of the exploration.

· A rationale explaining why the exploration was chosen.

· Sensible conclusions based on the outcomes of your findings.

· Discussion points and explanations that make sense to an audience of your peers.

· Arguments that quickly get to the point.

· Graphs &/or tables &/or diagrams &/or spreadsheets which are clearly labelled and easily understood.

· Graphs with a key if needed, axes labelled appropriately.

· Broken your exploration up into sections with suitable headings.

· Included page numbers and bibliography.

B: Mathematical Communication

To achieve B4 your mathematical communication must be relevant, appropriate and consistent throughout.

Appropriate means that the mathematical presentation will be as required. This means:

a) Good use of an equation editor if you use any equations. No computer notation.

b) Having all tables clearly labelled.

c) Having all graphs with a title and with axes labelled.

d) Having defined all variables and explained all mathematical terms.

e) Rounding to an appropriate degree and demonstrating an understanding of the level of accuracy you are using.

Relevant means that you choose the mathematical presentation that is required to communicate the topic aim. Including graphs or tables which are not necessary in addressing your aim is not relevant.

Consistent means that your mathematical presentation does not change throughout the exploration (e.g. changing from y to Y) and is consistently appropriate.

Criteria B student checklist – have you done the following?

Have you

· Used correct mathematical language throughout the exploration

· Used correct mathematical notation and symbols throughout the exploration – **NO** computer or GDC notation is acceptable ie *, ^, /

· Used the approximately equal to sign when rounding numbers

· Defined terms and variables **before** they are used in your working and referred to in your explanations

· Used multiple forms of mathematical representation such as formulae, diagrams, tables, charts, graphs, and models as appropriate.

C: Personal Engagement

To achieve C3 there must be evidence of outstanding personal engagement. You can show **significant** personal engagement [C2] by:

a) Thinking independently – looking at a problem from a unique perspective. Very formulaic investigations or ones which are very similar to textbook problems are to be avoided.

b) Showing your personal interest – explain why you are so interested in this exploration and maintain your personal voice throughout when commenting, explaining and reflecting.

c) Exploring the topic – this is an exploration and not a summary of other people's work. Try to explore from different perspectives or using different mathematical methods.

d) Trying to learn some new mathematics or trying to apply mathematics you already know to new contexts.

e) If you are doing an investigation which requires data then try to collect this data yourself. Show how you were engaged in this process.

You can show **abundant** personal engagement [C3] by doing a significant amount of the ideas expressed above, to a high quality and showing a creative approach to meeting the aim of the exploration.

Criteria C student checklist – have you done the following?

Have you

· Clearly outlined or identified a personal connection with your exploration in your rationale with reasons.

· Researched relevant background information for your exploration using external sources.

· Related the exploration to your personal experiences

· Used self-generated data if possible.

· Drawn conclusions that relate back to the original rationale and your interest shown in the introduction.

· Drawn conclusions, made reflections and arguments that show originality of thought

· Used Maths that you have learnt yourself, or used and applied mathematics from the syllabus in a new context.

· Made suitable improvements using the feedback given for the draft version of the exploration.

D: Reflection

To achieve D3 you must show substantial evidence of critical reflection.
You can show **meaningful reflection** [D2] by:

a) Commenting on what you have learnt and how it links to your initial aim.

b) Providing reflection on results – put them into real life context where possible. What are the implications of your results?

c) Comparing different approaches to solving a problem and addressing the accuracy of these methods.

d) Looking at the strengths, weaknesses and limitations of your investigation and providing ideas for further study.

To show **substantial critical reflection** [D3] you should be doing all of the above, to a very high standard and doing it throughout the exploration.

Criteria D student checklist – have you done the following?

Have you

· Made comments on the results **and** the process of the exploration

· Made comments on how the results and the process could be improved.

· Made comments within the context of the subject matter of the exploration.

· Made sensible recommendations based on your findings.

· Drawn conclusions that refer back to the aim and rationale of your exploration.

· Made comments on possible limitations of your exploration.

· Considered and discussed future implications of the results of the exploration that could perhaps lead to further explorations.

E: Use of Mathematics

Standard Level students:

To achieve at least E3 the mathematics must be **commensurate** with the level of the Standard Level course. This means it should be using mathematics from the Standard Level syllabus or mathematics of a similar level. The maths used should be **relevant** to the stated aim of the investigation. If complicated maths is used when much simpler maths could have been used instead then this is not relevant. Choosing to do a Pearson's Product correlation calculation when you have already drawn a scatter graph and shown no correlation is not relevant.

To achieve at least E5 students need to **demonstrate** good knowledge and understanding. The IB defines demonstrate as, "to make clear by reasoning or evidence, illustrating with examples or practical application." Each step of any calculation should be clearly explained and any thought processes written down to demonstrate that the student does clearly understand the mathematics they are using.

To achieve E6 students need for all the mathematics to be **correct** and to have demonstrated a **thorough knowledge** and understanding of their topic. Students who choose an exploration that goes beyond the scope of SL maths usually struggle to demonstrate thorough understanding (and often good understanding).

Higher Level students:

To achieve E3 the mathematics used by HL students must be **correct.** It must also be **relevant** to the exploration and **commensurate** with the level of the course. [contained within the syllabus, see above].

To achieve E5 students must demonstrate **sophistication** or **rigour** and **thorough** knowledge.

Sophisticated mathematics should be that which is specifically on the HL rather than SL syllabus, or SL maths which has been extended beyond what an SL student would be reasonably expected to produce.

Rigour requires a high level of logical thought in all calculations and justification for claims made (or proof presented).

To achieve E6 students must show both sophistication **and** rigour and also **precise** mathematics – every calculation must be correct and relevant levels of accuracy considered throughout.

Choosing a topic

1) Choose something you are interested in finding out more about. What course do you want to do at university? What are your other IB subjects – could you link with them? What are your hobbies and interests? But remember this is a **maths investigation** – the topic you choose has to be of a level relevant to your IB Maths course.

2) Choose a topic that you are able to explore – and ideally that has a question you can seek the answer to. Do not simply produce a summary of some maths. You need to be able to look for an answer to a question – which then can allow you to reflect on later.

3) Choose a topic that is at the right level of maths for the course. Too easy and you will not get a good grade on criteria E, too hard and you could lose marks on criteria A and E.

4) Try to avoid topics that have been done hundreds of times before – you will find it much easier to impress the moderator with an interesting and novel investigation rather than one he/she has seen many times before.

Main Topic areas

1) Modelling functions

2) Pure Maths

3) Geometry

4) Stats and probability

5) Calculus

6) Linking with other subjects

Modelling functions

The main idea

Fit a curve to a function or describe a real life process through equations.

Why is this topic a good idea?

This topic can allow you to link functions, transforming graphs and calculus together.

Examples:

1) Traffic flow: How maths can model traffic on the roads.

2) Impact Earth – what would happen if an asteroid or meteorite hit the Earth?

3) Modelling infectious diseases – how we can use mathematics to predict how diseases like measles will spread through a population

4) Modelling Zombies – How do zombies spread? What is your best way of surviving the zombie apocalypse? Surprisingly maths can help!

5) Modelling music with sine waves – how we can understand different notes by sine waves of different frequencies. Listen to the sounds that different sine waves make.

6) The Gini Coefficient – How to model economic inequality

7) Maths of Global Warming – Modeling Climate Change – Using Desmos to model the change in atmospheric Carbon Dioxide.

8) Modelling radioactive decay – the mathematics behind radioactivity decay, used extensively in science.

9) Torus – solid of revolution: A torus is a donut shape which introduces some interesting topological ideas.

10) Projectile motion: Studying the motion of projectiles like cannon balls is an essential part of the mathematics of war.

11) Batman and Superman maths – how to use Wolfram Alpha to plot graphs.

Pure Maths

The main idea:

Pure maths is often involved in finding patterns and rules that help us understand sequences and numbers.

Why is this topic a good idea?

This topic allows you to experience what "real" mathematicians do with their time - take a problem and then explore how it can be solved. Can it be solved using different methods? Which method is best?

Examples:

1) Plotting the Mandelbrot set: The stunning graphics of Mandelbrot and Julia Sets are generated by complex numbers.

2) Waging war with maths - an investigation into hollow squares and hollow cubes.

3) The Mordell equation: How to find the differences between square and cube numbers.

4) Ramanujan's taxi cab number - explore the links between number theory, graphs and group theory.

5) Chinese remainder theorem. This is a puzzle that was posed over 1500 years ago by a Chinese mathematician. It involves understanding the modulo operation.

6) Perfect Numbers: Perfect numbers are the sum of their factors (apart from the last factor). ie 6 is a perfect number because 1 + 2 + 3 = 6.

7) Time travel to the future: Investigate how traveling close to the speed of light allows people to travel "forward" in time relative to someone on Earth.

8) RSA code – the most important code in the world? How all our digital communications are kept safe through the properties of primes.

9) The Chinese Remainder Theorem: This is a method developed by a Chinese mathematician Sun Zi over 1500 years ago to solve a numerical puzzle.

10) Square triangular numbers - How to use number theory and computing to find numbers which are both square and triangular numbers.

Geometry

The main idea:

Geometry looks at some form of describing images through lines and points. A part of mathematics over 2000 years old.

Why is this topic a good idea?

This topic allows you to combine investigation techniques alongside graphical software.

Examples:

1) Non-Euclidean geometries: This allows us to “break” the rules of conventional geometry – for example, angles in a triangle no longer add up to 180 degrees.

2) Mandelbrot set and fractal shapes: Explore the world of infinitely generated pictures and fractional dimensions. Also Sierpinksi triangles: a fractal design.

3) Soap bubbles, wormholes and catenoids - explore minimal surfaces and the shapes they create.

4) Graphically understanding complex roots – have you ever wondered what the complex root of a quadratic actually means graphically?

5) Circular inversion – what does it mean to reflect in a circle? A great introduction to some of the ideas behind non-euclidean geometry.

6) Graphing Stewie from Family Guy. How to use graphic software to make art from equations.

7) The Coastline Paradox – how we can measure the lengths of coastlines, and uses the idea of fractals to arrive at fractional dimensions.

8) The Folium of Descartes. This is a nice way to link some maths history with studying an interesting function.

9) Measuring the Distance to the Stars. Maths is closely connected with astronomy – see how we can work out the distance to the stars.

10) Euler’s 9 Point Circle. This is a lovely construction using just compasses and a ruler.

Stats and Probability 1

The main idea:

Stats and probability help us understand a variety of other subjects and are useful for predictions. These topics below have scope for in-depth investigation.

Why is this topic a good idea?

This allows you to link mathematical techniques with other subjects in the sciences.

Examples:

1) Quantum mechanics - statistical universe. Look at how probability underpins reality.

2) The Martingale system - investigate how a centuries old system of doubling down on losing bets is still used in currency trading today.

3) Using Chi Squared to crack codes – Chi squared can be used to crack Vigenere codes which for hundreds of years were thought to be unbreakable. Unleash your inner spy!

4) Are you psychic? Use the binomial distribution to test your ESP abilities.

5) Reaction times – are you above or below average? Model your data using a normal distribution.

6) Modelling volcanoes – look at how the Poisson distribution can predict volcanic eruptions, and perhaps explore some more advanced statistical tests.

7) Could Trump win the next election? How the normal distribution is used to predict elections.

8) Gambler's fallacy: A good chance to investigate misconceptions in probability and probabilities in gambling. Why does the house always win?

9) Birthday paradox: The birthday paradox shows how intuitive ideas on probability can often be wrong. How many people need to be in a room for it to be at least 50% likely that two people will share the same birthday? Find out!

Stats and Probability 2

The main idea:

Stats and probability help us understand a variety of other subjects and are useful for predictions. These topics below lend themselves to simpler stats investigations.

Why is this topic a good idea?

This allows you to link mathematical techniques with other subjects in the sciences

1) Is there a correlation between the digit ratio and maths ability? Studies suggest there is a correlation between digit ratio and academic ability. Is this true?

2) Is there a correlation between GDP and life expectancy? Run the Gapminder graph to show the changing relationship between GDP and life expectancy over the past few decades.

3) Is there a correlation between stock prices of different companies? Use Google Finance to collect data on company share prices.

4) Is there a correlation between Premier League wages and league positions?

5) Are the IB maths test scores normally distributed? IB test scores approximately fit bell curves. Investigate how the scores from different IB subjects compare.

6) Investigation into the distribution of word lengths in different languages. The English language has an average word length of 5.1 words. How does that compare with other languages?

7) Do bilingual students have a greater memory recall than non-bilingual students? Studies have shown that bilingual students have better "working memory" – does this include memory recall?

8) Are a sample of student heights normally distributed? We know that adult population heights are normally distributed – what about student heights?

9) Which times tables do students find most difficult to learn? – Are some calculations like 7×8 harder than others? Why?

Calculus

The main idea:

Calculus is a very powerful tool for understanding the world around us

Why is this topic a good idea?

Allows you to look at a mixture of modelling and pure maths together and to use a number of calculus techniques.

Examples:

1) The Monkey and the Hunter – How to Shoot a Monkey – Using Newtonian mathematics to decide where to aim when shooting a monkey in a tree.

2) How to Design a Parachute – looking at the physics behind parachute design to ensure a safe landing!

3) Galileo: Throwing cannonballs off The Leaning Tower of Pisa – Recreating Galileo's classic experiment, and using maths to understand the surprising result.

4) Area optimisation – an investigation.

5) Bullet projectile motion experiment – using Tracker software to model the motion of a bullet.

6) Radiocarbon dating – understanding radioactive decay allows scientists and historians to accurately work out something's age – whether it be from thousands or even millions of years ago.

7) Intergalactic space travel and time dilation – Essential knowledge for future astronauts.

8) Lagrange points – and how these are used for satellites. Investigate some rocket science!

9) Creating 3D shapes using volume of revolutions - volume of a rugby ball.

10) Volume optimisation of cuboids - an investigation.

Linking with other subjects and areas in maths

The main idea:

Maths is used across many different areas – explore some of these.

Why is this topic a good idea?

Allows you to look at another subject using mathematical ideas. Good if you like another subject (perhaps if you want to do this at university).

Examples:

1) Mathematical methods in economics – maths is essential in both business and economics – explore some economics based maths problems.

2) Genetics – Look at the mathematics behind genetic inheritance and natural selection.

3) Medical data mining – Explore the use and misuse of statistics in medicine and science.

4) Designing bridges – Mathematics is essential for engineers such as bridge builders – investigate how to design structures that carry weight without collapse.

5) Solving maths problems using computers – computers are really useful in solving mathematical problems. Here are some examples solved using Python.

6) Simulating a football season using maths.

7) Codes and ciphers: ISBN codes and credit card codes are just some examples of how codes are essential to modern life. Maths can be used to both make these codes and break them.

8) Chinese postman problem – This is a problem from graph theory – how can a postman deliver letters to every house on his streets in the shortest time possible?

9) Telephone numbers - explore some graph theory and how it can link to calculus.

Useful programs/sites

1) The **Census at School** website is a fantastic source of secondary data to use. If you go to the random data generator you can download up to 200 questionnaire results from school children around the world on a number of topics.

2) If you like football you can also find a lot of football stats on the Who Scored website. This gives you data on things like individual players' shots per game, pass completion rate etc.

3) The **World Bank** has a huge data bank – which you can search by country or by specific topic. You can compare life-expectancy rates, GDP, access to secondary education, spending on military, social inequality, how many cars per 1000 people and much much more.

4) Gapminder is another great resource for comparing development indicators – you can plot 2 variables on a graph (for example urbanisation against unemployment, or murder rates against urbanisation) and then run them over a number of years. You can also download Excel speadsheets of the associated data.

5) Wolfram Alpha is one of the most powerful maths and statistics tools available – it has a staggering amount of information that you can use.

6) Plotly is a great visual graphic site – you can create visually interesting infographics and analyse data from hundreds of other sources.

7) TSM – the Technology for Secondary Mathematics is something of an internet dinosaur – but has a great deal of downloadable data files on everything from belly-button ratios to lottery number analysis and baby weights.

8) Google Public Data – an enormous source for public data, which is displayed graphically and can be searched.

9) Nationmaster – another huge site with pretty much any statistic and data comparing countries. Currently they have 19 million data points!

10) Desmos – a great online graphing site

11) Geogebra – another very powerful graphing application (also does 3D)

12) Tracker software will allow you to track data from a video and then produce graphs.

IB HL/SL Exploration Initial Submission Sheet

First Choice Topic (what question will you answer?):

Outline of how your exploration could develop:

Does this topic allow you to reflect on your results? How?

What is the SL/HL maths (or equivalent) contained in this topic?

Why did you choose this topic?

Second Choice Topic:

Outline of how your exploration could develop:

Does this topic allow you to reflect on your results? How?

What is the SL/HL maths (or equivalent) contained in this topic?

Why did you choose this topic?

Draft submission checklist:

1. Are you handing in what you believe to be a finished exploration?

2. Does your introduction set out a plan for what will happen in the investigation?

3. Does your introduction explain why you have chosen your topic and demonstrate a personal engagement?

4. Is your exploration easy to follow for a fellow IB student?

5. Do you label all graphs and define all mathematical terms? Do you use an equation editor throughout?

6. Do you have a bibliography and citations throughout?

7. Do you have a conclusion which discusses what you have learnt and discusses ideas for further study?

8. Have you reflected throughout the investigation as to the meaning of your results? What do they show? How could they be improved? Don't just arrive at results and move on!

9. Have you produced an individual piece of work? This should be personalised throughout.

10. Have you answered the question you set out to investigate? Can you reflect on how successful you have been? Have you altered your investigation to reflect your results?

Some last minute points to check on your Maths IA:

Criteria A – Communication

Have you

· An introduction explaining the exploration
· An aim outlining the purpose of the exploration
· A rationale explaining why the exploration was chosen
· Sensible conclusions based on the outcomes of your findings
· Discussion points and explanations that make sense to an audience of your peers
· Arguments that **quickly** get to the point
· Graphs &/or tables &/or diagrams &/or spreadsheets which are clearly labelled and easily understood
· Graphs with a key if needed, axes labelled appropriately
· Broken your exploration up into sections with suitable headings
· Included page numbers
· Included a bibliography

Criteria B – Mathematical Presentation

Have you

· Used correct mathematical language throughout the exploration
· Used correct mathematical notation and symbols throughout the exploration – **NO** computer or GDC notation is acceptable ie *, ^, /
· Used the approximately equal to sign when rounding numbers (≈)
· Defined terms and variables **before** they are used in your working and referred to in your explanations
· Used multiple forms of mathematical representation such as formulae, diagrams, tables, charts, graphs and models as appropriate

Criteria C – Personal Engagement

Have you

· Clearly outlined or identified a personal connection with your exploration in your rationale with reasons
· Researched relevant background information for you exploration using external sources
· Related the exploration to your personal experiences
· Used self-generated data if possible
· Drawn conclusions that relate back to the original rationale and your interest shown in the introduction
· Drawn conclusions, made reflections and arguments that show originality of thought
· Used Maths that goes beyond the level of the course with understanding
· Made suitable improvements using the feedback given for the draft version of the exploration

Criteria D – Reflection

Have you

· Made comments on the results **and** the process of the exploration
· Made comments on how the results and the process could be improved
· Made comments within the context of the subject matter of the exploration
· Made sensible recommendations based on your findings
· Drawn conclusions that refer back to the aim and rationale of your exploration
· Made comments on possible limitations of your exploration
· Considered and discussed future implications of the results of the exploration that could perhaps lead to further explorations

Criteria E – Use of Mathematics

Have you

· Used relevant mathematics commensurate with the level of the course
· Made sure all the mathematics used is correct
· Demonstrated a thorough knowledge and understanding of the mathematics

Common mistakes

A: Communication strand.

You should have a clear aim which allows you to investigate in a logical manner and then reflect and conclude by completing this aim.

“My aim is to investigate projectiles in football” is a **poor aim**. It is vague and doesn’t naturally allow a good investigative flow.

“My aim is to compare Ronaldo’s free-kicks with my free-kicks to become a better footballer” is a **much better aim**. This investigation will have a clear flow, lots of opportunities for personal engagement, reflection and could be successfully completed.

C: Personal engagement strand.

Choose a topic which allows you to express yourself personally. Whilst other students may have done the same topic you, yours should still be sufficiently personalised to be unique.
A topic which will **struggle** to show good personal engagement:
“I will look at the correlation between GDP and test scores for 10 countries.”
[Data will be secondary, difficult to show significant personal angle, basic correlation investigations are very common and formulaic].
A topic which **could show** good personal engagement:
“Designing a children’s swing bridge for our school’s playground”
[Opportunity for personal data collection, opportunity to produce a unique investigation, opportunity for use of some self-taught maths, opportunity for links with DT in making a model].

B: Mathematical Communication strand.

Poor communication:

f(x) = 3x/2 +4*sinx

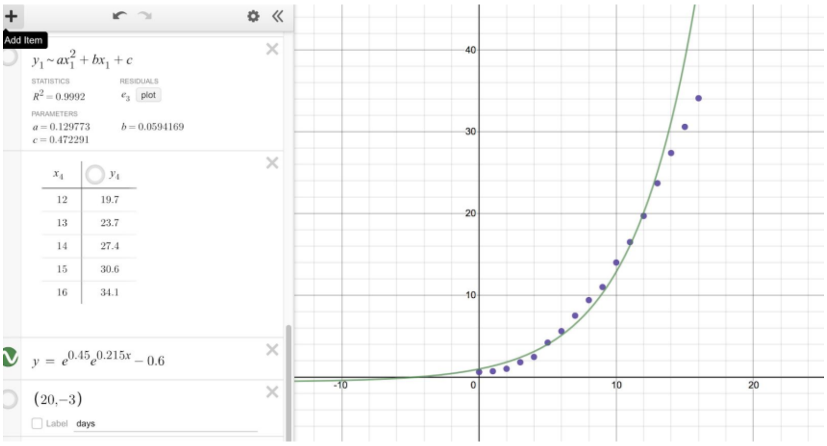
When X = 1, I get 4.9 metres.

Good communication:

t is the time in seconds. *h(t)* is the height in metres. Angles are in radians.

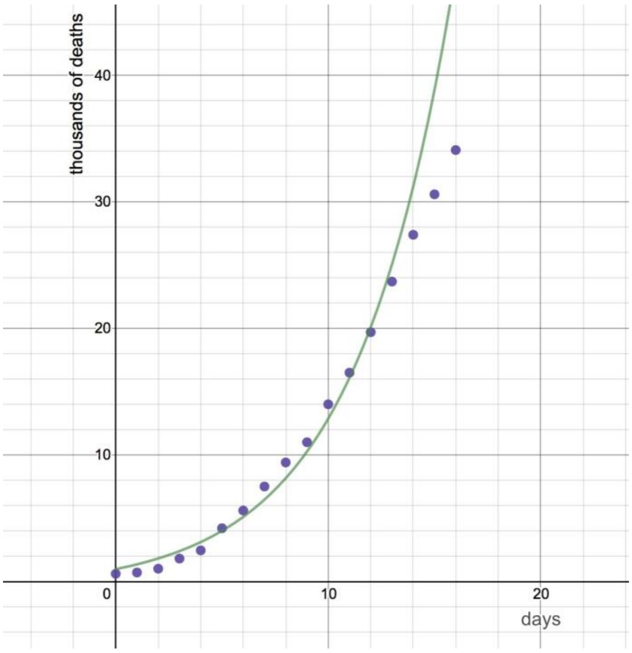
$$h(t) = \frac{3t}{2} + 4sin(t)$$
$$h(1) = \frac{3(1)}{2} + 4sin(1)$$
$h(1) \approx$ 4.87 (to 3 significant figures).

Poor communication: Poor screen capture, hard to read, axes not labelled, no title



Good communication: [axes labelled, graph given a title, domain restricted]

Figure 1: Plotting the number of deaths in a Covid-19 outbreak



D: Reflection strand.

Poor reflection:

I did linear regression and got an r value of 0.91. This shows there is strong positive correlation.

Good meaningful reflection:

I did linear regression and got an *r* value of 0.91. This shows there is strong positive correlation between height and weight. As weight is my dependent variable this shows that as someone's height increases, their weight also increases. My linear regression line of best fit is:

$w = 10h + 40$

w : *weight in kg*
h : *height in metres*

This means that for every extra metre in height we would expect someone to weigh an extra 10kg. However this equation is only valid for male adults, and because of the data we used the domain should be restricted to $h \geq 1.5$. We can see that without restricting the domain we would have a prediction of someone with no height being 40kg. This clearly makes no sense. I can use my equation to predict the weight of someone who is 1.7m tall:

$w = 10(1.7) + 40$

$w = 57$

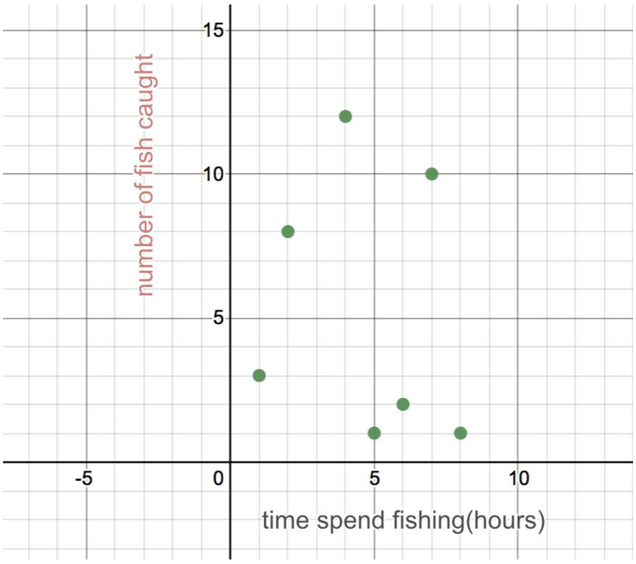
This seems like a reasonable prediction because I measured my classmate who is 1.7m tall and their weight was 63kg. This gives me confidence that my model is accurate.

E: Use of mathematics strand.

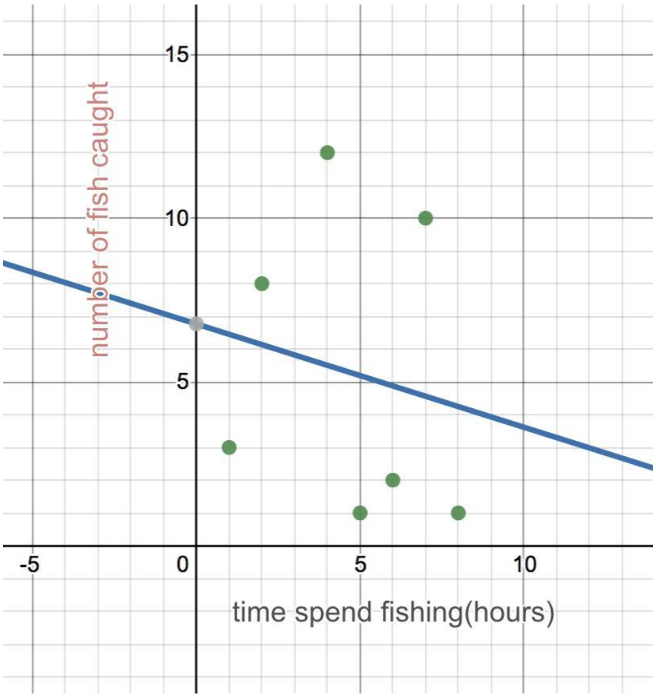
For SL students I'd strongly advise you not to choose mathematics that is more difficult than the level of the SL course. Students rarely are able to demonstrate good understanding and hence get a poor mark. HL students can potentially go a bit beyond the HL syllabus in terms of difficulty level - but it should not be much beyond.

Statistics projects frequently perform poorly in the E category - to demonstrate good understanding you need to show you understand why you're using the test you're using, what assumptions this test relies on, your sampling considerations etc. The most common low scoring stats investigation is a simple correlation between 2 variables where students first draw a scatter graph, note that there is weak or no correlation and then do a Pearson's product calculation. If the scatter graph already shows weak or no correlation then it is not relevant to do this.

Poor mathematical understanding:



I plotted the number of fish caught and time spent fishing. From my scatter graph I can see there is no correlation or very weak correlation. I then used Pearson's Product formula to find that $r \approx -0.175$ *(3sf).* I then found the line of best fit:

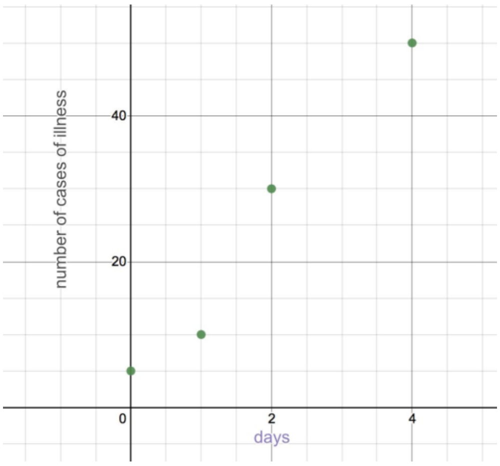


I will now use the line of best fit to predict how many fish will be caught after 15 hours. Etc.

Why does this show poor mathematical understanding?

Firstly there are only 7 data points - so any conclusions drawn from this are going to be weak at best. The student draws a scatter graph [this will achieve E2 for relevant maths], but because the scatter graph shows weak or no correlation then the following maths to use the Pearson's Product formula is not relevant. It is also not good understanding to use a line of best fit for such poorly fitting data, and not relevant in this case to extrapolate beyond the data range as this model suggests the more time you spend fishing the less you will catch!

Poor mathematical understanding:



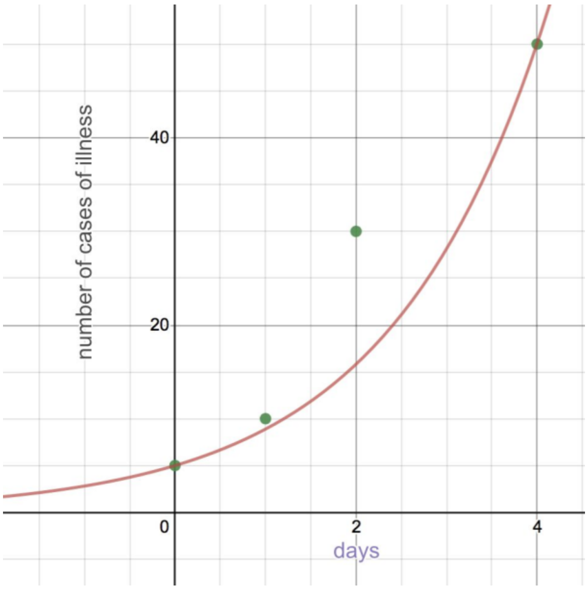
I plotted the graph above and tried to fit an exponential curve to the data points. The graph needs to pass through (0,5) and (4,50), so I can make the following equations:

$5 = ae^{b(0)}$

$50 = ae^{b(4)}$

Solving these give:

y = $5e^{0.576x}$



I also did an exponential regression using desmos to check the validity of an exponential model and got:

y = $9.39e^{0.425x}$

This gave an R^2 value of 0.912. This is close to 1 which shows that an exponential model is valid.

Why does this show poor mathematical understanding?

There are only 4 data points to use, the student chooses the first and last data point without justification. The exponential model is also chosen without justification - when clearly a linear model would be a better fit. The student does not show their method (and hence does not show understanding) of how to solve the simultaneous equations. $R2$ is not explained and does not give evidence that an exponential model is valid for this data.

Teacher marking

I thought I'd add some of my own thoughts with regards to teacher marking - other teachers may disagree, but this is what I have found helpful when moderating:

1. Teachers providing **evidence** for levels awarded on a cover-sheet. Just copying and pasting from the criteria points adds no extra value compared to just writing a level down.

Contrast:

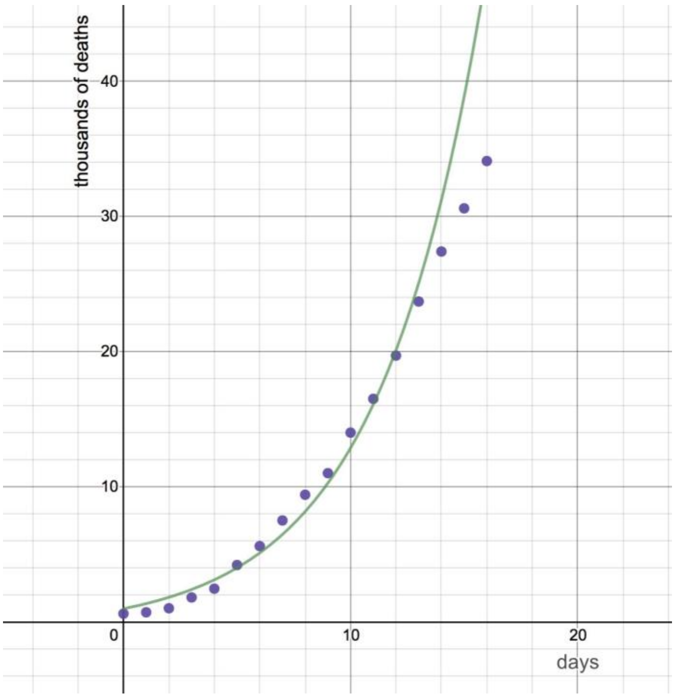
C2: There was meaningful engagement.

C2: The student is a very talented Grade 7 pianist who is passionate about music and so really was keen to do an exploration that allowed them to investigate the mathematics of music. I liked the personal element of trying to understand this topic from the perspective of how a computer would interpret music. This was a novel approach. The student did an interesting experiment to try to determine the pitch of a tuning fork and it was good to see cross-curricular work with the Physics department here . The student also did a lot of additional work on this topic to try to understand Fourier transforms – which required a lot of background reading. They showed real engagement in this topic throughout the whole process.

2. Teachers providing **annotation** on the work itself to highlight criteria evidence, to show that the mathematics has been checked (and is correct) or to highlight errors.

Example 1:

Figure 1: Plotting the number of deaths in a Covid-19 outbreak



B+ Axes are labelled, graph has a title. Domain restricted.

Example 2

I plotted the graph above and tried to fit an exponential curve to the data points. The graph needs to pass through (0,5) and (4,50), so I can make the following equations:

$5 = ae^{b(0)}$

$50 = ae^{b(4)}$

Solving these give:

$y = 5e^{0.576x}$

A- Steps are not clearly explained

E+ Maths has been checked and is correct.

Example 3

I did linear regression and got an *r* value of 0.91. This shows there is a strong positive correlation between height and weight. As weight is my dependent variable this shows that as someone's height increases, their weight also increases. My linear regression line of best fit is:

$w = 10h + 40$

w : *weight in kg*

h : *height in metres*

B+ Variables defined

This means that for every extra metre in height we would expect someone to weigh an extra 10kg. However this equation is only valid for male adults, and because of the data we used the domain should be restricted to $h \geq 1.5$. We can see that without restricting the domain we would have a prediction of someone with no height being 40kg. This clearly makes no sense. I can use my equation to predict the weight of someone who is 1.7m tall:

$w = 10(1.7) + 40$

$w = 57$

D+ Meaningful attempt to reflect on real life intepretation

This seems like a reasonable prediction because I measured my classmate who is 1.7m tall and their weight was 63kg. This gives me confidence that my model is accurate.